

Exercise 20

Compute Δy and dy for the given values of x and $dx = \Delta x$. Then sketch a diagram like Figure 5 showing the line segments with lengths dx , dy , and Δy .

$$y = x - x^3, \quad x = 0, \quad \Delta x = -0.3$$

Solution

Compute the derivative of y .

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(x - x^3) \\ &= 1 - 3x^2 \end{aligned}$$

Consequently, the differential of $y = x - x^3$ is

$$dy = (1 - 3x^2) dx,$$

so when $x = 0$ and $\Delta x = dx = -0.3$,

$$dy = [1 - 3(0)^2](-0.3) = -0.3$$

$$\Delta y = y(0 - 0.3) - y(0) = [(0 - 0.3) - (0 - 0.3)^3] - [(0) - (0)^3] = -0.273.$$

The function is plotted below along with its tangent line at $x = 0$.

